



Oxford Cambridge and RSA

**Monday 20 May 2019 – Afternoon**

**AS Level Further Mathematics A**

**Y535/01 Additional Pure Mathematics**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g\text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1** In decimal (base 10) form, the number  $N$  is 15 260.
- (a) Express  $N$  in binary (base 2) form. [1]
- (b) Using the binary form of  $N$ , show that  $N$  is divisible by 7. [2]
- 2** (a) The convergent sequence  $\{a_n\}$  is defined by  $a_0 = 1$  and  $a_{n+1} = \sqrt{a_n} + \frac{4}{\sqrt{a_n}}$  for  $n \geq 0$ .  
Calculate the limit of the sequence. [1]
- (b) The convergent sequence  $\{b_n\}$  is defined by  $b_0 = 1$  and  $b_{n+1} = \sqrt{b_n} + \frac{k}{\sqrt{b_n}}$  for  $n \geq 0$ , where  $k$  is a constant.  
Determine the value of  $k$  for which the limit of the sequence is 9. [3]
- 3** The non-zero vectors  $\mathbf{x}$  and  $\mathbf{y}$  are such that  $\mathbf{x} \times \mathbf{y} = \mathbf{0}$ .
- (a) Explain the geometrical significance of this statement. [2]
- (b) Use your answer to part (a) to explain how the line equation  $\mathbf{r} = \mathbf{a} + t\mathbf{d}$  can be written in the form  $(\mathbf{r} - \mathbf{a}) \times \mathbf{d} = \mathbf{0}$ . [2]
- 4** The sequence  $\{u_n\}$  is defined by  $u_1 = 1$  and  $u_{n+1} = 2u_n + n^2$  for  $n \geq 1$ .  
Determine  $u_n$  as a function of  $n$ . [8]

- 5 The tetrahedron  $T$ , shown below, has vertices at  $O(0, 0, 0)$ ,  $A(1, 2, 2)$ ,  $B(2, 1, 2)$  and  $C(2, 2, 1)$ .

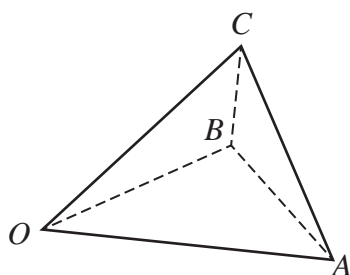


Diagram not drawn to scale

Show that the surface area of  $T$  is  $\frac{1}{2}\sqrt{3}(1 + \sqrt{51})$ . [8]

- 6 (a) Determine all values of  $x$  for which  $16x \equiv 5 \pmod{101}$ . [4]

(b) Solve

(i)  $95x \equiv 6 \pmod{101}$ , [2]

(ii)  $95x \equiv 5 \pmod{101}$ . [2]

- 7 You are given the set  $S = \{1, 5, 7, 11, 13, 17\}$  together with  $\times_{18}$ , the operation of multiplication modulo 18.

(a) Complete the Cayley table for  $(S, \times_{18})$  given in the Printed Answer Booklet. [4]

(b) Prove that  $(S, \times_{18})$  is a group. (You may assume that  $\times_{18}$  is associative.) [3]

(c) Write down the order of each element of the group. [2]

(d) Show that  $(S, \times_{18})$  is a cyclic group. [1]

(e) (i) Give an example of a non-cyclic group of order 6. [1]

(ii) Give one reason why your example is structurally different to  $(S, \times_{18})$ . [1]

Turn over for question 8

- 8 The motion of two remote controlled helicopters  $P$  and  $Q$  is modelled as two points moving along straight lines.

Helicopter  $P$  moves on the line  $\mathbf{r} = \begin{pmatrix} 2 + 4p \\ -3 + p \\ 1 + 3p \end{pmatrix}$  and helicopter  $Q$  moves on the line  $\mathbf{r} = \begin{pmatrix} 5 + 8q \\ 2 + q \\ 5 + 4q \end{pmatrix}$ .

The function  $z$  denotes  $(PQ)^2$ , the square of the distance between  $P$  and  $Q$ .

- (a) Show that  $z = 26p^2 + 81q^2 - 90pq - 58p + 90q + 50$ . [3]
- (b) Use partial differentiation to find the values of  $p$  and  $q$  for which  $z$  has a stationary point. [4]
- (c) With the aid of a diagram, explain why this stationary point must be a minimum point, rather than a maximum point or a saddle point. [2]
- (d) Hence find the shortest possible distance between the two helicopters. [2]

The model is now refined by modelling each helicopter as a sphere of radius 0.5 units.

- (e) Explain how this will change your answer to part (d). [2]

### END OF QUESTION PAPER

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